

Relativity and Quantum Field Theory

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ABSTRACT. Relativistic quantum field theories (RQFTs) are invariant under the action of the Poincaré group, the symmetry group of Minkowski spacetime. Non-relativistic quantum field theories (NQFTs) are invariant under the action of the symmetry group of a classical spacetime; *i.e.*, a spacetime that minimally admits absolute spatial and temporal metrics. This essay is concerned with cashing out two implications of this basic difference: First, under a Received View of particles, RQFTs do not admit particle interpretations. I will argue that this view is motivated by non-relativistic intuitions and hence should be abandoned. In particular, just as classical spacetimes can be related to Minkowski spacetime by appropriate limits, NQFTs can be related to RQFTs under similar limits. And just as the classical (non-relativistic) concept of simultaneity is a degenerate version of the relativistic concept under the appropriate limiting relation, the Received View's notion of particle can be viewed as a classical (non-relativistic) degenerate version of a relativistic notion, under the appropriate limit. This suggests that the Received View is being informed by out-dated physics. Second, the relations between RQFTs and NQFTs also suggest that routes to quantum gravity are more varied than is typically acknowledged. The second half of this essay is concerned with mapping out some of this conceptual space.

1. Introduction
2. Galilei-invariant Quantum Field Theories
3. Newtonian Quantum Gravity
4. Particle Interpretations
5. Intertheoretic Relations
6. Conclusion

1. Introduction

Non-relativistic quantum field theories (NQFTs) are employed as empirically successful approximations in condensed matter physics in descriptions of many-body quantum systems with (ideally) infinite degrees of freedom in which energies and propagation speeds are typically low (compared with the speed of light). High-energy particle physics, on the other hand, is considered to be the domain of fully relativistic (*i.e.*, Poincaré-invariant) quantum field theories (RQFTs). These are theories that describe phenomena with infinite degrees of freedom at high energies and propagation speeds approaching the speed of light. In both condensed matter physics and high-energy particle physics, the gravitational force is ignored. Our current best theory of the latter is a relativistic non-quantum field theory (*viz.*, general relativity), and while there are examples of NQFTs that do incorporate gravity, the search is still on for a fully relativistic quantum theory of gravity (be it a field theory or otherwise).

This essay is primarily concerned with fleshing out the intertheoretic relations between NQFTs and RQFTs, and how these relations influence approaches to formulating and interpreting a relativistic quantum theory of gravity. It does this by first reviewing

NQFTs in the absence of gravity (Section 2), and then reviewing an example of an NQFT that incorporates gravity (Section 3). Section 2 reviews Galilei-invariant quantum field theories (GQFTs); *i.e.*, quantum field theories invariant under the action of the extended Galilei group, the symmetry group of non-relativistic quantum mechanics (Lévy-Leblond 1967). In such theories, Haag's theorem does not apply, and while a version of the Reeh-Schlieder theorem is valid, it does not preclude the existence of local number operators as it does in the relativistic context. These results indicate that, in the absence of gravity, and under a received view of particles, particle interpretations find a less troubled home in NQFTs than they do in RQFTs (Halvorson & Müger 2007, Fraser preprint). Section 3 reviews a non-relativistic theory of quantum gravity in Newton-Cartan spacetime due to Christian (1997). It is similar in construction to RQFTs in curved spacetimes, except part of its spacetime structure is dynamic and quantized, and its symmetry group is an extension of the non-relativistic Maxwell group. The latter entails that it is not plagued by the family of conceptual problems associated with unitarily inequivalent representations of the canonical (anti-) commutation relations, as are RQFTs, both free and interacting, in curved spacetimes (see, *e.g.*, Ruetsche 2002). These results suggest that the presence of gravity in principle does not disrupt the tranquil homelife of standard particle interpretations in NQFTs. Section 4 mounts an argument against this received view of particles: I argue that, to the extent that the received view is compatible with NQFTs and not with RQFTs, it derives from pre-relativistic intuitions that should be jettisoned in the relativistic context. Section 5 undertakes the task of relating NQFTs, both in the presence and the absence of gravity, to RQFTs and to other theories, both of particles and fields, classical and quantum, in the presence and the absence of gravity. What emerges is a map of the relations between some of the fundamental theories in physics, including the as-yet-to-be formulated, fully relativistic quantum theory of gravity.

Sample References

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